TECHNICAL REPORT

Incorporating Gyromass Lumped Parameter Models (GLPMs) in OpenSees

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Introduction

Lumped parameter model with gyro-mass element (GLPMs) have been proposed recently and the accuracy of these models have been verified over the conventional methods to represent frequency dependent impedance functions of soil-foundation systems. In this report, the methodology to incorporate GLPMs in OpenSees framework is described. OpenSees is a finite element application to simulate structural and geotechnical systems. Currently, the gyromass element is not available in the OpenSees framework; hence a transformation of GLPMs is necessary which is also described in detail in the following chapters.

1.1 Base system

The base system is composed of a gyromass $\bar{M}$ and a spring K and a dashpot C arranged in parallel as shown in Fig. 1(a). This model contains gyromass as a component hence this model cannot be directly incorporated in OpenSees. Hence, this model is transformed as shown in Fig. 1(b) by fixing a node after which the gyromass becomes a simple mass. The detail of transformation method is explained in Appendix-A.

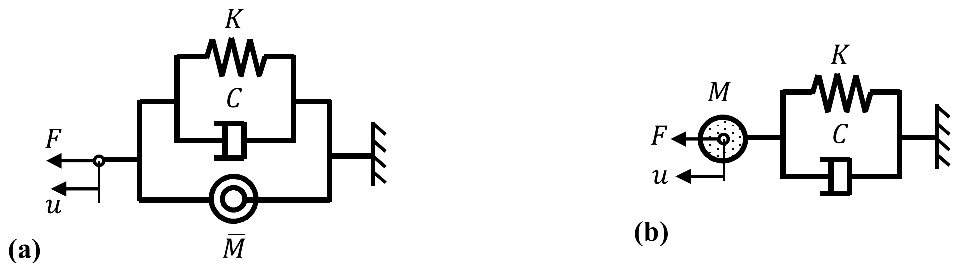

Fig. 1. Base system of type II model (a); base system after transformation (b)

Model

The model consists of two nodes, and spring and damper are connected in parallel with each other. Node 2 (the right node) is fixed and the sinusoidal load is applied in Node 1.

```
wipe
# Build model
model basicBuilder -ndm 1 -ndf 1

#Define nodes
node 1  0
node 2  0

#Base system
set k 1; #Spring
set c 1; # Damper
set m 1; #mass

# Materials Parameter & Element Connectivity
# Horizontal Direction
uniaxialMaterial Elastic 1 $k
uniaxialMaterial Viscous 2 $c 1.
      uniaxialMaterial Parallel 3 1 2
      element zeroLength 1 1 2 -mat 3 -dir 1

# Define masses
mass 1 $m

# Define Boundary Conditions
fix 2 1

# Define Time Series
timeSeries Trig 1 0.0 200.0 100.0; # 100.0 sec time period

# Define load
pattern Plain 1 1 {
load 1 1.0
}
```

Analysis

The transient analysis objects are defined next. The integrator for the dynamic analysis is Newmark's method that considers constant average acceleration in one time step of analysis, $\gamma = 0.5$ and $\beta = 0.25$. This choice is unconditionally stable for linear problems.

```
integrator Newmark 0.5 [expr 1./4.]
test EnergyIncr 1.0e-12 50 1
algorithm Linear
numberer Plain
constraints Plain
system BandGeneral
analysis Transient
```

Output Specification

For the analysis we'll record the displacement at node 1.

```
recorder Node -file d_Node2f.out -time -node 1 -dof 1 disp; # Resp Dis
```

Perform the Analysis

After the objects for the model, analysis and output have been defined to perform the analysis using 200000 time increments with a time step $\delta t = 0.001$.

```
analyze 200000 0.001
```

Results

The analysis is repeated for the desired frequency period and outputs are analyzed to find out the maximum amplitude and phase lag and impedance functions are plotted against dimensionless frequency. The impedance functions provided by Saitoh in his paper and the impedance functions calculated by using OpenSees overlapped with each other as shown in Fig. 2.

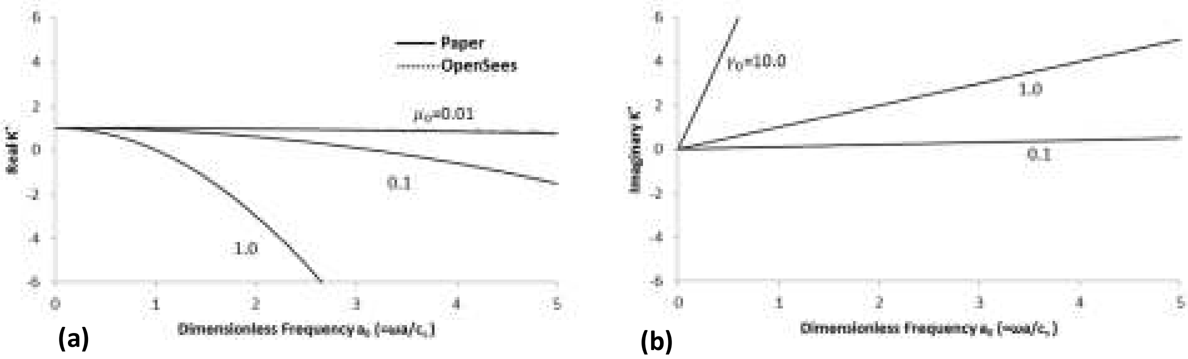

Fig. 2. Variation of impedance functions with coefficients μ_0 and $\gamma_0$in the base system (*K*=1.0). (a) Real part (stiffness); (b) imaginary part (damping).

1.2 Core system

The core system is composed of a gyromass $\bar{M}$ and a dashpot C is connected in parallel and a spring K is connected in series as shown in Fig. 3(a). Again the model is transformed by making a node fixed as shown in Fig. 3(b) by which gyromass becomes a simple mass.

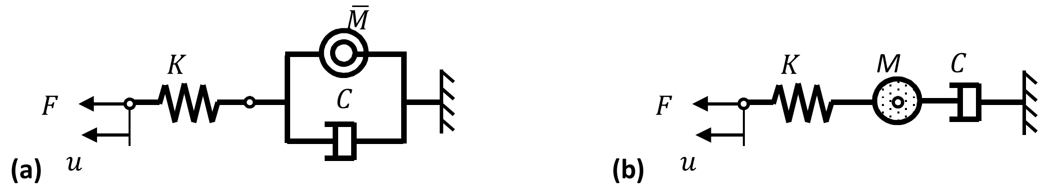

Fig. 3. Core system of the type II model (a); the core system after transformation (b).

Model

The model consists of three nodes, and spring and damper are connected in series. Node 1 is fixed and the sinusoidal load is applied in Node 3.

```
wipe
# Build model
model basicBuilder -ndm 1 -ndf 1

#Define nodes
node 1  0
node 2  0
node 3  0

#Base system
set k 1.; #Spring
set c 5.; # Damper
set m 1.; #mass

# Materials Parameter & Element Connectivity
# Horizontal Direction
uniaxialMaterial Elastic 1 $k
      element zeroLength 1 2 3 -mat 1 -dir 1
uniaxialMaterial Viscous 2 $c 1.
      element zeroLength 2 1 2 -mat 2 -dir 1

# Define masses
mass 2 $m
# Boundary Conditions
fix 1 1


#Define TimeSeries
timeSeries Trig 1 0.0 150.0 100.0; #100.0 sec time period

#Define Load
pattern Plain 1 1 {
load 3 1.0
}
```

Analysis

The transient analysis objects are defined next. The integrator for the dynamic analysis is Newmark's method that considers constant average acceleration in one time step of analysis, $\gamma = 0.5$ and $\beta = 0.25$.

```
integrator Newmark 0.5 [expr 1./4.]
test EnergyIncr 1.0e-12 20 1
algorithm Linear
numberer Plain
constraints Plain
system BandGeneral
analysis Transient
```

Output Specification

For the analysis we'll record the displacement at node 3.

```
recorder Node -file d_Node2f.out -time -node 3 -dof 1 disp; # Resp Dis
```

Perform the Analysis

After the objects for the model, analysis and output has been defined we perform the analysis using 150000 time increments with a time step $\delta t = 0.001$.

```
analyze 150000 0.001
```

Results

This analysis is run for a range of frequency and each output file is analyzed to find out the maximum amplitude and the phase lag. Again impedance functions are calculated and plotted against dimensionless frequency as shown in Fig. 4. These results are compared with the results provided by Saitoh in his paper, in which both results are overlapped.

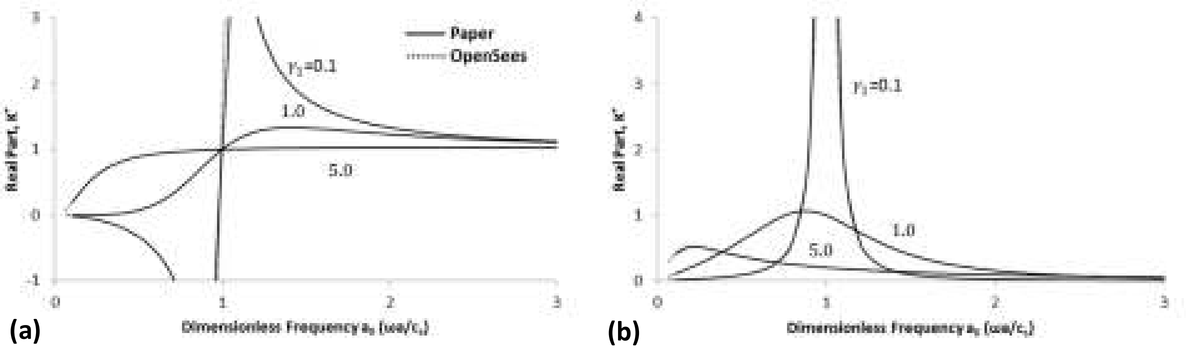

Fig. 4. Variation of impedance functions with several values of the coefficient γ_1 in the core system of the Type II model (k=1.0 and μ_1=1.0). (a) Real part (stiffness); (b) imaginary part (damping).

1.3 GLPMs

The total system is composed of a base system and a number of core systems arranged in parallel. Saitoh presented an example in his paper which is modeled by type II model having a base system and three core systems. The same example is taken to make a model in OpenSees as shown in Fig. 5(a). The transformed model as shown in Fig. 5(b) is used to construct a model in OpenSees. The summarized properties of the model are presented in Table I which is also presented by Saitoh. This type II model can be constructed in rotational direction by replacing the mass by mass moment of inertia, dashpots by rotational dashpots and spring by rotational springs of the type II model.

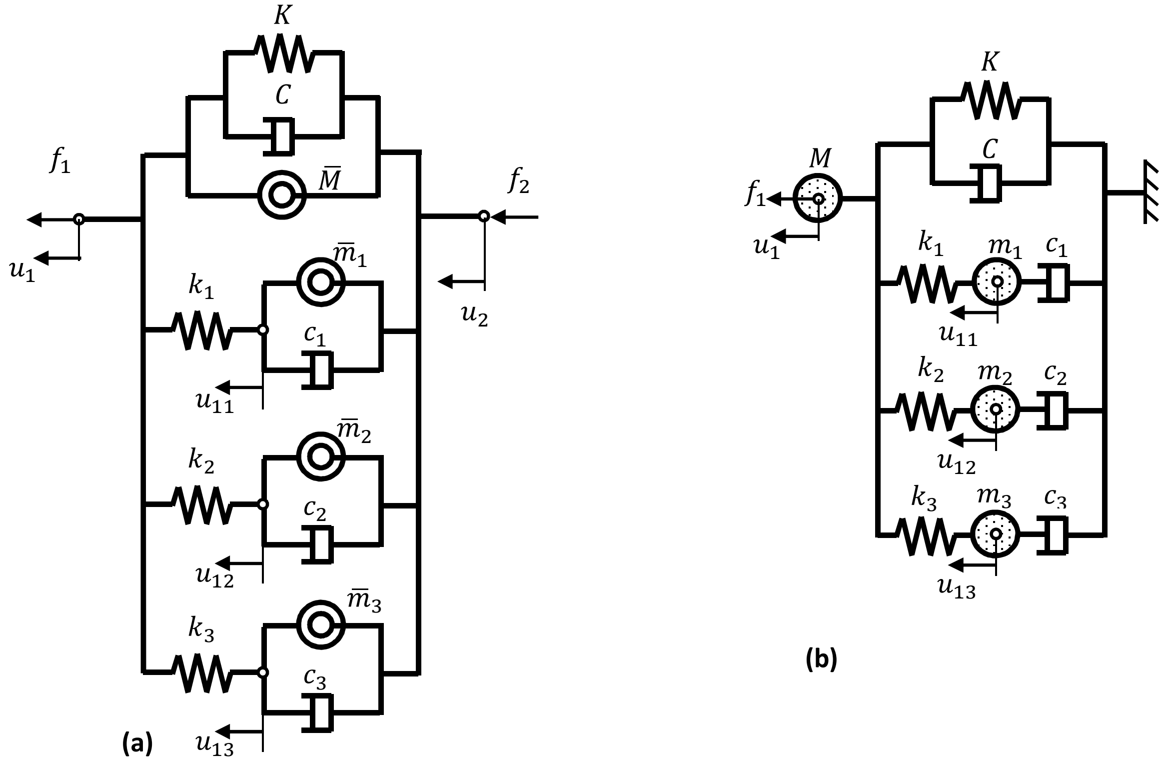

Fig. 5. Type II model having a base system and three core system (a); the same model after transformation (b).

Table I. Properties of Type II model.

Horizontal Direction			
Base System		Core System 2	
Spring K	$2.40\text{x}10^5$ (kN/m)	Spring k_2	$1.56\text{x}10^5$ (kN/m)
Damper C	$8.16\text{x}10^3$ (kNs/m)	Damper c_2	$3.74\text{x}10^3$ (kNs/m)
Gyromass $\overline{M}$	$2.40\text{x}10^2$ (kNs^2/m)	Gyromass $\overline{m}_2$	$5.46\text{x}10^2$ (kNs^2/m)
Core System 1		Core System 3	
Spring k_1	$8.64\text{x}10^4$ (kN/m)	Spring k_3	$1.20\text{x}10^5$ (kN/m)
Damper c_1	$2.59\text{X}10^3$ (kNs/m)	Damper c_3	$2.16\text{x}10^3$ (kNs/m)
Gyromass $\overline{m}_1$	$5.18\text{X}10^2$ (kNs^2/m)	Gyromass $\overline{m}_3$	$6.00\text{x}10^1$ (kNs^2/m)
Rotational Direction			
Base System		Core System 2	
Spring $\widetilde{K}$	$6.87\text{x}10^8$ (kNm/rad)	Spring $\widetilde{k}_2$	$3.44\text{x}10^7$ (kNm/rad)
Damper $\widetilde{C}$	$2.20\text{x}10^6$ (kNms/rad)	Damper $\widetilde{c}_2$	$8.93\text{x}10^5$ (kNms/rad)
Gyromass $\overline{J}$	$6.87\text{x}10^4$ (kNms^2/rad)	Gyromass $\overline{J}_2$	$5.15\text{x}10^4$ (kNms^2/rad)
Core System 1		Core System 3	
Spring $\widetilde{k}_1$	$5.50\text{x}10^6$ (kNm/rad)	Spring $\widetilde{k}_3$	$6.87\text{x}10^6$ (kNm/rad)
Damper $\widetilde{c}_1$	$1.10\text{x}10^5$ (kNms/rad)	Damper $\widetilde{c}_3$	$5.50\text{x}10^4$ (kNms/rad)
Gyromass $\overline{J}_1$	$2.47\text{x}10^4$ (kNms^2/rad)	Gyromass $\overline{J}_3$	$4.81\text{x}10^3$ (kNms^2/rad)

1.3.1 Horizontal Direction
Model

```
# Define the model
model basicBuilder -ndm 1 -ndf 1

# Define the nodes
node 1 0
node 2 0
node 3 0
node 4 0
node 5 0

# Define the variables
#Base system
set k 2.4e+5; #Spring
set c 8.16e+3; # Damper
set m 2.4e+2; #mass
#core system 1
set k1 8.64e+4; #Spring
set c1 2.592e+3;# Damper
set m1 5.184e+2; #mass
#core system 2
set k2 1.56e+5; #Spring
set c2 3.744e+3; # Damper
set m2 5.46e+2; #mass
#core system 3
set k3 1.2e+5; #Spring
```

```
set c3 2.16e+3; # Damper
set m3 6.0e+1; #mass

# Set the boundary conditions
fix 1 1

# Define the masses at the nodes
mass 2 $m
mass 3 $m1
mass 4 $m2
mass 5 $m3

# Materials Parameter & Element Connectivity
uniaxialMaterial Elastic 1 $k
uniaxialMaterial Viscous 2 $c 1.
      uniaxialMaterial Parallel 3 1 2
      element zeroLength 1 1 2 -mat 3 -dir 1
uniaxialMaterial Elastic 4 $k1
      element zeroLength 2 3 2 -mat 4 -dir 1
uniaxialMaterial Viscous 5 $c1 1.
      element zeroLength 3 1 3 -mat 5 -dir 1
uniaxialMaterial Elastic 6 $k2
      element zeroLength 4 4 2 -mat 6 -dir 1
uniaxialMaterial Viscous 7 $c2 1.
      element zeroLength 5 1 4 -mat 7 -dir 1
uniaxialMaterial Elastic 8 $k3
      element zeroLength 6 5 2 -mat 8 -dir 1
uniaxialMaterial Viscous 9 $c3 1.
      element zeroLength 7 1 5 -mat 9 -dir 1


# Define the timeseries
timeSeries Trig 1 0.0 50.0 10.0; # 10.0 is time period

# Define the load pattern
pattern Plain 1 1 {
load 2 1.0
}
```

Analysis

```
integrator Newmark 0.5 [expr 1./4.]
test EnergyIncr 1.0e-12 20 1
algorithm Linear
numberer Plain
constraints Plain
system BandGeneral
analysis Transient
```

Output Specification

```
recorder Node -file d_Node2f.out -time -node 2 -dof 1 disp; # Resp Dis
```

Perform Analyis

```
analyze 500000 0.0001
```

Results

The impedance functions presented in paper by Saitoh and obtained by using OpenSees is overlapped as shown in Fig. 6.

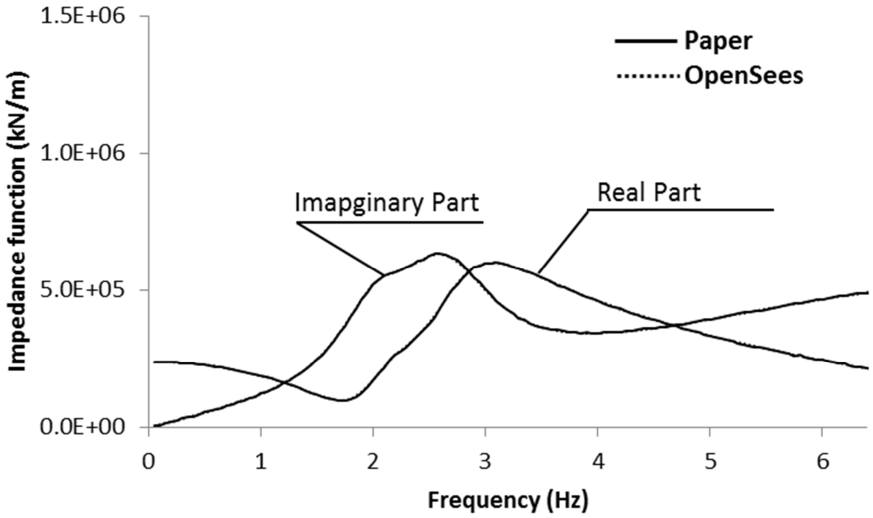

Fig. 6. Comparisons of impedance function of pile group in horizontal direction from OpenSees with Saitoh's paper.

1.3.2 Rotational Direction

Model

```
# Define the model
model basicBuilder -ndm 2 -ndf 3

# Define the nodes
node 1 0 0
node 2 0 0
node 3 0 0
node 4 0 0
node 5 0 0

# Define the variables
#Base system
set k 6.87e+8; #Spring
set c 2.20e+6; # Damper
set m 6.87e+4; #mass
#core system 1
set k1 5.50e+6; #Spring
set c1 1.10e+5;# Damper
set m1 2.47e+4; #mass
#core system 2
set k2 3.44e+7; #Spring
set c2 8.93e+5; # Damper
set m2 5.15e+4; #mass
#core system 3
set k3 6.87e+6; #Spring
set c3 5.50e+4; # Damper
set m3 4.81e+3; #mass
```

```
# Define the boundary conditions
fix 1 1 1 1
fix 2 1 1 0
fix 3 1 1 0
fix 4 1 1 0
fix 5 1 1 0

# Define masses
mass 2 0 0 $m
mass 3 0 0 $m1
mass 4 0 0 $m2
mass 5 0 0 $m3

# Materials Parameter & Element Connectivity
uniaxialMaterial Elastic 1 $k
uniaxialMaterial Viscous 2 $c 1.
      uniaxialMaterial Parallel 3 1 2
      element zeroLength 1 1 2 -mat 3 -dir 6
uniaxialMaterial Elastic 4 $k1
      element zeroLength 2 3 2 -mat 4 -dir 6
uniaxialMaterial Viscous 5 $c1 1.
      element zeroLength 3 1 3 -mat 5 -dir 6
uniaxialMaterial Elastic 6 $k2
      element zeroLength 4 4 2 -mat 6 -dir 6
uniaxialMaterial Viscous 7 $c2 1.
      element zeroLength 5 1 4 -mat 7 -dir 6
uniaxialMaterial Elastic 8 $k3
      element zeroLength 6 5 2 -mat 8 -dir 6
uniaxialMaterial Viscous 9 $c3 1.
      element zeroLength 7 1 5 -mat 9 -dir 6

# Define Timeseries
timeSeries Trig 1 0.0 50.0 10.0

# Define Load
pattern Plain 1 1 {
load 2  0.0 0.0 1.0
}
```

Analysis

```
integrator Newmark 0.5 [expr 1./4.]
test EnergyIncr 1.0e-12 20 1
algorithm Linear
numberer Plain
constraints Plain
system BandGeneral
analysis Transient
```

Output Specification

```
recorder Node -file d_Node2f.out -time -node 2 -dof 3 disp; # Resp Dis
```

Perform Analysis

```
analyze 500000 0.0001
```

Results

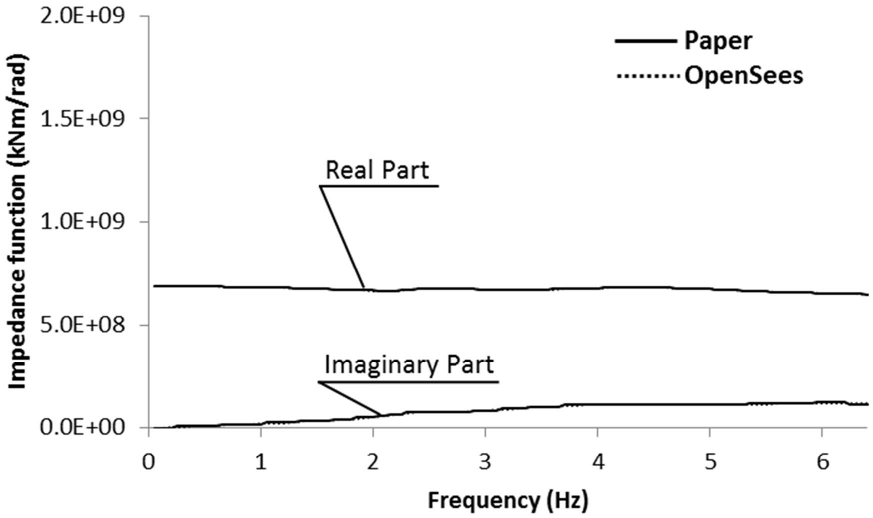

Fig. 7. Comparisons of impedance function of pile group in rotational direction from OpenSees with Saitoh's paper.

1.4 Example

This example is taken from the Saitoh's paper. Here a SDOF system is supported by an array of two by four vertical, cylindrical piles of diameter d = 1.2 m and length L = 18 m which are embedded in layered soil. The pile-to-pile distance in longitudinal direction is 8 m and the distance in the transverse direction is 5.8 m. The Young's modulus, mass density, and Poisson's ratio of the piles are $E_p = 2.5 \times 10^7\ kN/m^2$, $\rho_p = 2.5\ ton/m^3$, and $\nu_p = 0.25$, respectively. Each pile cap is rigidly connected with a footing consisting of stiff beams, imposing unique displacements u_f and $\theta_f$on the footing in the horizontal and rotational directions, respectively. The masses of superstructure and footing are $m_s = 800\ ton$, and $m_f = 350\ ton$, respectively. The mass moment of inertia of the footing is $J_f = 15{,}000\ tm^2$. The stiffness and damping coefficient of SDOF system are assumed to be $k_s = 256{,}000\ kN/m$ and $c_s = 143\ kNs/m$, respectively. The height of SDOF system is $H_s = 8\ m$. The properties of the soils are shown in Fig. 8.

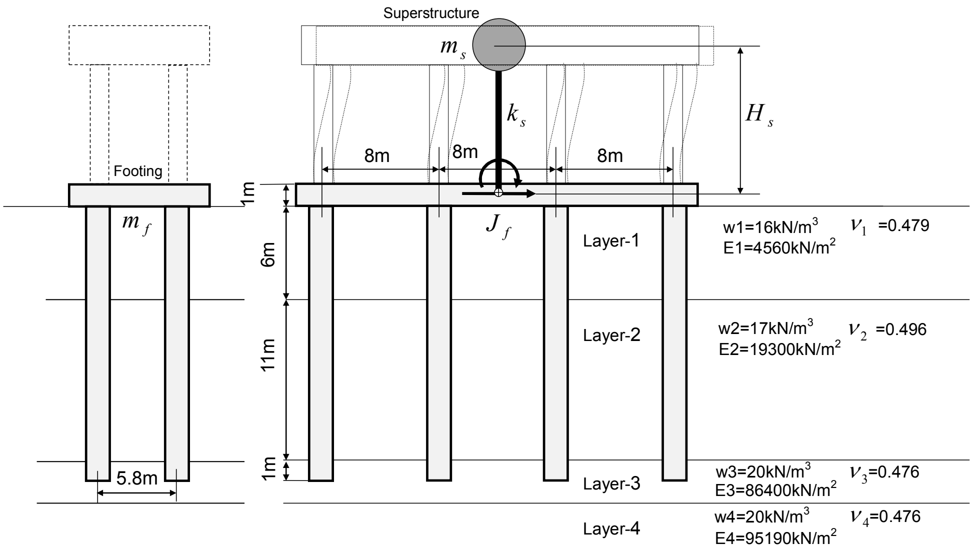

Fig. 8 Soil–pile system studied. Layer-*i* denotes *i*-th soil layer. The unit weight, the modulus of elasticity, Poisson's ratio, and the damping ratio of the *i*-th soil layer are denoted as w_i, E_i,ν_i, and h_i, respectively.

Above described problem is modeled using GLPMs in horizontal and rotational direction as shown in Fig. 9.

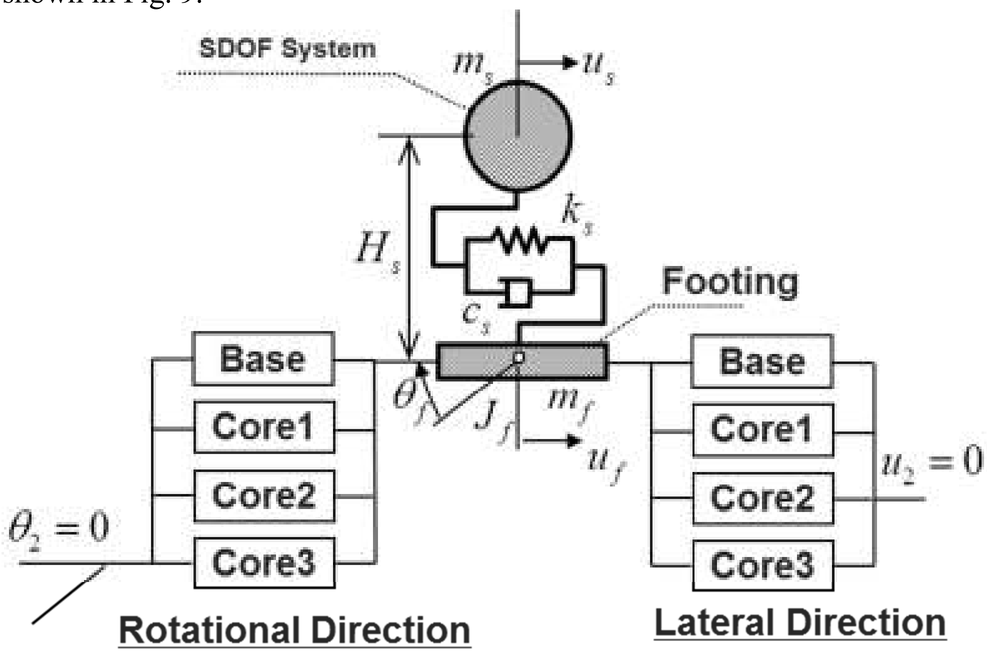

Fig. 9. Numerical model representing total structural system

1.4.1 Transfer Functions

Model

```
# Build model
model basicBuilder -ndm 2 -ndf 3

# set the height of the structure and assumed width and depth
set Hs 8.0
set b 1.0
set d 1.0

#Define nodes
node 1  0 0
node 2  0 0
node 3  0 0
node 4  0 0
node 5  0 0
node 7  0 0
node 8  0 0
node 9  0 0
node 10 0 0
node 12 0 $Hs

# Define the parameters value
#Horizontal direction
#Base system
set k 2.4e+5; #Spring
set c 8.16e+3; # Damper
set m 2.4e+2; #mass
#core system 1
set k1 8.64e+4; #Spring
set c1 2.592e+3;# Damper
set m1 5.184e+2; #mass
#core system 2
set k2 1.56e+5; #Spring
set c2 3.744e+3; # Damper
set m2 5.46e+2; #mass
#core system 3
set k3 1.2e+5; #Spring
set c3 2.16e+3; # Damper
set m3 6.0e+1; #mass

#Rotational direction
#Base system
set rk 6.87e+8; #Spring
set rc 2.20e+6; # Damper
set rm 6.87e+4; #mass
#core system 1
set rk1 5.50e+6; #Spring
set rc1 1.10e+5;# Damper
set rm1 2.47e+4; #mass
#core system 2
set rk2 3.44e+7; #Spring
set rc2 8.93e+5; # Damper
set rm2 5.15e+4; #mass
#core system 3
set rk3 6.87e+6; #Spring
```

```
set rc3 5.50e+4; # Damper
set rm3 4.81e+3; #mass

#SuperStructure
set sk 256000.0; #Spring
set sc 1431.0; # Damper
set sm 800.0; #mass
set mf 350.0; #mass of foundation
set jf 15000.0; #Jf

# Materials Parameter & Element Connectivity
# Horizontal Direction
uniaxialMaterial Elastic 1 $k
uniaxialMaterial Viscous 2 $c 1.
      uniaxialMaterial Parallel 3 1 2
      element zeroLength 1 1 2 -mat 3 -dir 1
uniaxialMaterial Elastic 4 $k1
      element zeroLength 2 1 3 -mat 4 -dir 1
uniaxialMaterial Viscous 5 $c1 1.
      element zeroLength 3 3 2 -mat 5 -dir 1
uniaxialMaterial Elastic 6 $k2
      element zeroLength 4 1 4 -mat 6 -dir 1
uniaxialMaterial Viscous 7 $c2 1.
      element zeroLength 5 4 2 -mat 7 -dir 1
uniaxialMaterial Elastic 8 $k3
      element zeroLength 6 1 5 -mat 8 -dir 1
uniaxialMaterial Viscous 9 $c3 1.
      element zeroLength 7 5 2 -mat 9 -dir 1

# Rotational Direction
uniaxialMaterial Elastic 11 $rk
uniaxialMaterial Viscous 12 $rc 1.
      uniaxialMaterial Parallel 13 11 12
      element zeroLength 11 7 1 -mat 13 -dir 6
uniaxialMaterial Elastic 14 $rk1
      element zeroLength 12 8 1 -mat 14 -dir 6
uniaxialMaterial Viscous 15 $rc1 1.
      element zeroLength 13 7 8 -mat 15 -dir 6
uniaxialMaterial Elastic 16 $rk2
      element zeroLength 14 9 1 -mat 16 -dir 6
uniaxialMaterial Viscous 17 $rc2 1.
      element zeroLength 15 7 9 -mat 17 -dir 6
uniaxialMaterial Elastic 18 $rk3
      element zeroLength 16 10 1 -mat 18 -dir 6
uniaxialMaterial Viscous 19 $rc3 1.
      element zeroLength 17 7 10 -mat 19 -dir 6

# Superstructure
# Calculate Young's modulus and second moement of inertia
set I [expr $Hs*$d*$d*$d/12.]; # moment of inertia of the section
set E [expr $sk*$Hs*$Hs*$Hs/3./$I]; # Young's modulus of elasticity

geomTransf Linear 1
element elasticBeamColumn 21 1 12 $Hs $E $I  1

# Define Damping
region 1 -ele 21 -eleRange 20 25 -rayleigh 0 [expr $sc/$sk] 0 0
```

```
# Define masses
mass 1  [expr $m+$mf] 0 [expr $jf+$rm]
mass 3  $m1 0  0
mass 4  $m2 0  0
mass 5  $m3 0  0

mass 8  0 0 $rm1
mass 9  0 0 $rm2
mass 10 0 0 $rm3

mass 12 $sm 0 0

# Boundary Conditions

fix 2  1 1 1; # Fixed in all direction
fix 3  0 1 1; # Horizontal Free
fix 4  0 1 1; # Horizontal Free
fix 5  0 1 1; # Horizontal Free

fix 7 1 1 1; # Fixed in all direction
fix 8 1 1 0; # Rotation Free
fix 9 1 1 0; # Rotation Free
fix 10 1 1 0; # Rotation Free

fix 12 0 1 0; #SuperStructure

# Define timeseries
timeSeries Trig 1 0.0 50.0 10.0

# Define Loads
pattern Plain 1 1 {
load 12 [expr $sm*(1.0)] 0.0 0.0
load 1 [expr $mf*(1.0)] 0.0 0.0
}
```

Analysis

```
integrator Newmark 0.5 [expr 1./4.]
test EnergyIncr 1.0e-12 50 1
algorithm Linear
numberer Plain
constraints Plain
system BandGeneral
analysis Transient
```

Define Outputs

```
recorder Node -file d_Node1f.out -time -node 1 -dof 1 disp; # Fdn Dis
recorder Node -file rd_Node1f.out -time -node 1 -dof 3 disp; # Fdn Rot
recorder Node -file d_Node12f.out -time -node 12 -dof 1 disp; # Sup Dis
recorder Node -file a_Node1f.out -time -node 1 -dof 1 accel; # Fdn acc
recorder Node -file ra_Node1f.out -time -node 1 -dof 3 accel; # Fdn Rot acc
recorder Node -file a_Node12f.out -time -node 12 -dof 1 accel; # Sup acc
```

Run the anlaysis

```
analyze 50000 0.001
```

Results

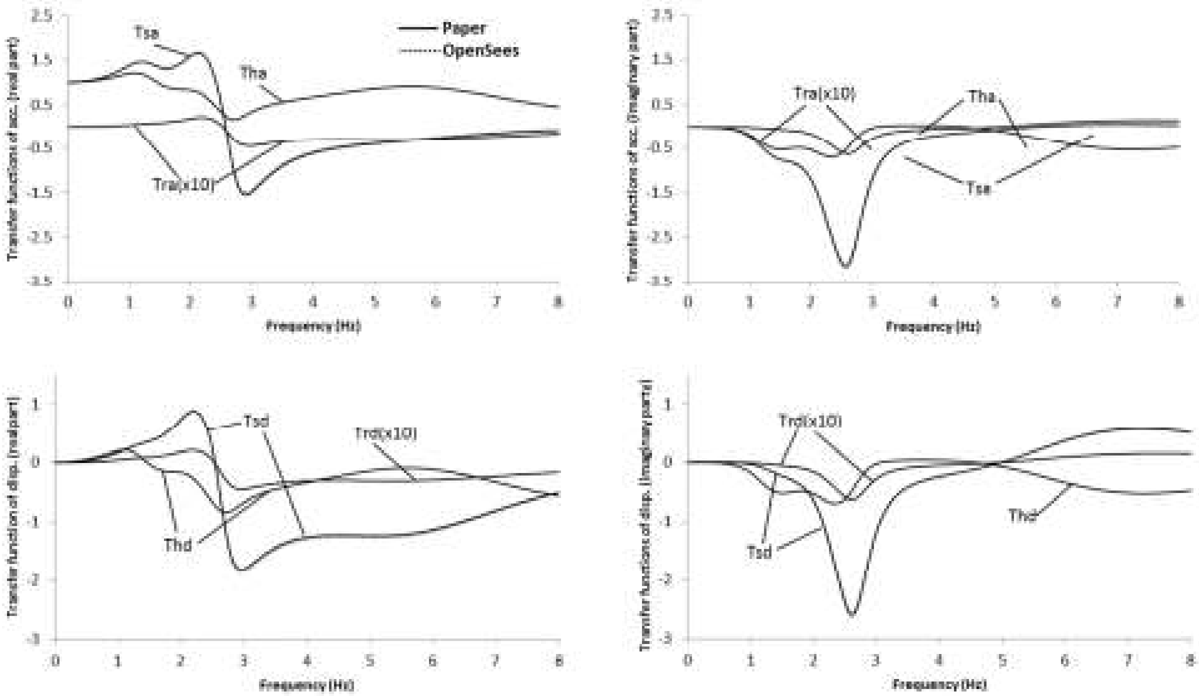

Fig. 10. Comparison of transfer functions obtained from OpenSees with Saitoh's paper.

1.4.2 Dynamic response of the structural system in the frequency domain.

Model

```
# Build model
model basicBuilder -ndm 2 -ndf 3

# set some parameters
set b 1.0
set d 1.0
set Hs 8.0

#Define nodes
node 1  0 0
node 2  0 0
node 3  0 0
node 4  0 0
node 5  0 0
node 7  0 0
node 8  0 0
node 9  0 0
node 10 0 0
```

```
node 12 0 $Hs

#Horizontal direction
# ---------------------

#Base system
set k 2.4e+5; #Spring
set c 8.16e+3; # Damper
set m 2.4e+2; #mass
#core system 1
set k1 8.64e+4; #Spring
set c1 2.592e+3;# Damper
set m1 5.184e+2; #mass
#core system 2
set k2 1.56e+5; #Spring
set c2 3.744e+3; # Damper
set m2 5.46e+2; #mass
#core system 3
set k3 1.2e+5; #Spring
set c3 2.16e+3; # Damper
set m3 6.0e+1; #mass

#Rotational direction
# ---------------------

#Base system
set rk 6.87e+8; #Spring
set rc 2.20e+6; # Damper
set rm 6.87e+4; #mass
#core system 1
set rk1 5.50e+6; #Spring
set rc1 1.10e+5;# Damper
set rm1 2.47e+4; #mass
#core system 2
set rk2 3.44e+7; #Spring
set rc2 8.93e+5; # Damper
set rm2 5.15e+4; #mass
#core system 3
set rk3 6.87e+6; #Spring
set rc3 5.50e+4; # Damper
set rm3 4.81e+3; #mass

#SuperStructure
# ---------------------
set sk 256000.0; #Spring
set sc 1431.0; # Damper
set sm 800.0; #mass
set mf 350.0; #mass of foundation
set jf 15000.0; #Jf

# Materials Parameter & Element Connectivity
# Horizontal Direction
uniaxialMaterial Elastic 1 $k
uniaxialMaterial Viscous 2 $c 1.
      uniaxialMaterial Parallel 3 1 2
      element zeroLength 1 1 2 -mat 3 -dir 1
uniaxialMaterial Elastic 4 $k1
```

```
      element zeroLength 2 1 3 -mat 4 -dir 1
uniaxialMaterial Viscous 5 $c1 1.
      element zeroLength 3 3 2 -mat 5 -dir 1
uniaxialMaterial Elastic 6 $k2
      element zeroLength 4 1 4 -mat 6 -dir 1
uniaxialMaterial Viscous 7 $c2 1.
      element zeroLength 5 4 2 -mat 7 -dir 1
uniaxialMaterial Elastic 8 $k3
      element zeroLength 6 1 5 -mat 8 -dir 1
uniaxialMaterial Viscous 9 $c3 1.
      element zeroLength 7 5 2 -mat 9 -dir 1

# Rotational Direction
uniaxialMaterial Elastic 11 $rk
uniaxialMaterial Viscous 12 $rc 1.
      uniaxialMaterial Parallel 13 11 12
      element zeroLength 11 7 1 -mat 13 -dir 6
uniaxialMaterial Elastic 14 $rk1
      element zeroLength 12 8 1 -mat 14 -dir 6
uniaxialMaterial Viscous 15 $rc1 1.
      element zeroLength 13 7 8 -mat 15 -dir 6
uniaxialMaterial Elastic 16 $rk2
      element zeroLength 14 9 1 -mat 16 -dir 6
uniaxialMaterial Viscous 17 $rc2 1.
      element zeroLength 15 7 9 -mat 17 -dir 6
uniaxialMaterial Elastic 18 $rk3
      element zeroLength 16 10 1 -mat 18 -dir 6
uniaxialMaterial Viscous 19 $rc3 1.
      element zeroLength 17 7 10 -mat 19 -dir 6

# Superstructure
# Calculate E and I
set I [expr $Hs*$d*$d*$d/12.]; # moment of inertia of the section
set E [expr $sk*$Hs*$Hs*$Hs/3./$I]; # Young's modulus of elasticity

geomTransf Linear 1
element elasticBeamColumn 21 1 12 $Hs $E $I  1

# Define Rayleigh damping for superstructure
region 1 -ele 21 -eleRange 20 25 -rayleigh 0 [expr $sc/$sk] 0 0

# Define masses
mass 1  [expr $m+$mf]  0 [expr $jf+$rm]
mass 3  $m1 0 0
mass 4  $m2 0 0
mass 5  $m3 0 0

mass 8   0 0 $rm1
mass 9   0 0 $rm2
mass 10  0 0 $rm3

mass 12 $sm  0 0


# Boundary Conditions
fix 1  0 0 0; # Horizontal & Rotation free
```

```
fix 2  1 1 1; # All Fixed
fix 3  0 1 1; # Horizontal Free
fix 4  0 1 1; # Horizontal Free
fix 5  0 1 1; # Horizontal Free

fix 7 1 1 1; # All Fixed
fix 8 1 1 0; # Rotation Free
fix 9 1 1 0; # Rotation Free
fix 10 1 1 0; # Rotation Free

fix 12 0 1 0; #SuperStructure

# Define Time Series
timeSeries Path 1 -dt 0.005 -filePath acc1_kobe.dat -factor 9.81

# Define Load
pattern Plain 1 1 {
load 12 [expr $sm*(-1.0)] 0.0 0.0
load 1 [expr $mf*(-1.0)] 0.0 0.0
}
```

Analysis

```
integrator Newmark 0.5 [expr 1./4.]
test EnergyIncr 1.0e-12 50 1
algorithm Linear
numberer Plain
constraints Plain
system BandGeneral
analysis Transient
```

Output Specification

```
recorder Node -file d_Node1f.out -time -node 1 -dof 1 disp; # Fdn Dis
recorder Node -file rd_Node1f.out -time -node 1 -dof 3 disp; # Fdn Rot Dis
recorder Node -file d_Node12f.out -time -node 12 -dof 1 disp; # Sup Dis
recorder Node -file a_Node1f.out -time -node 1 -dof 1 accel; # Fdn Acc
recorder Node -file ra_Node1f.out -time -node 1 -dof 3 accel; # Fdn Rot Acc
recorder Node -file a_Node12f.out -time -node 12 -dof 1 accel; # Sup Acc
```

Run the analysis

```
analyze 3200 0.005
```

Results

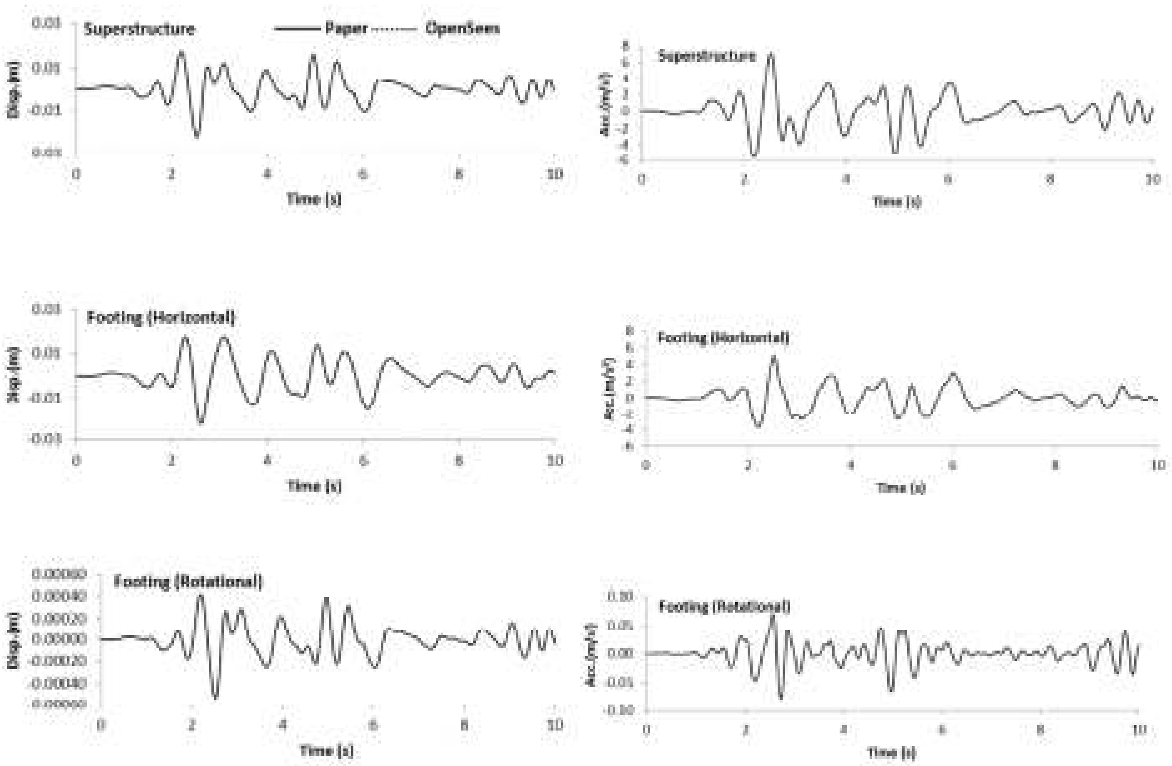

Fig. 11 . Time history responses of the superstructure and the footing in the horizontal and rotational directions when subjected to ground motion associated with 1940 El Centro NS by OpenSees.

The important point when analyzing the dynamic response of soil-foundation-superstructure systems subjected to ground motions with transformed GLPMs in OpenSees is that the external forces, which are compatible to the product of masses of the structures and the input acceleration (more accurately, the effective foundation input acceleration), are applied to the nodes in the systems. Note that any external forces corresponding to the equivalent mass transformed from gyromass in the GLPMs are not applied to the systems at all.
(see defined load in the example above)

Appendix-A

Model Transformation (base system)

The relation between forces and displacements at nodes of the base system is shown in Fig. A.1. Here the force f_1 and f_2 is applied at node 1 and node 2 respectively. The displacement at node 1 and node is represented by u_1 and u_2 respectively.

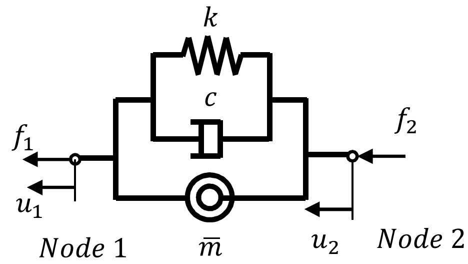

Fig. A.1. The relationship between forces and displacements at nodes of the base system

Let us write equation of motions for above described base system

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} \bar{m} & -\bar{m} \\ -\bar{m} & \bar{m} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{1}$$

Now if the node 2 of the base system is fixed, the force f_2 and displacement u_2 becomes zero and the Eq. (1) can be written as

$$f_1 = \bar{m}\ddot{u}_1 + c\dot{u}_1 + ku_1 \tag{2}$$

We know that the Eq. (2) is an equation of motion having a spring, dashpot connected in parallel and a mass at the end. Hence the gyromass of the base system of type II model can be converted to simple mass by fixing one node of the model. This system can be represented in a diagram as shown in Fig. A.2.

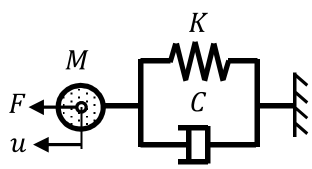

Fig. A.2. The transformed base system with mass, spring and damper.

Appendix-B

Model Transformation (core system)

The relationship between forces and displacements at nodes of core system of Type II model is shown in Fig. B.1. The core system is consist of three nodes. Forces f_1, f_2 and f_3 are applied in node 1, node 2 and node 3 respectively. u_1, u_2 and u_3 are the displacements at node 1, node 2 and node 3 respectively.

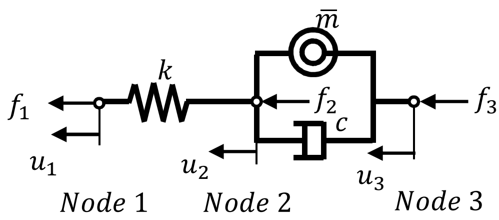

Fig. B.1. Relationship between force and displacements at the nodes of the core system.

Writing the equation of motion of this core system,

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{m} & -\bar{m} \\ 0 & -\bar{m} & \bar{m} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & c & -c \\ 0 & -c & c \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{Bmatrix} + \begin{bmatrix} k & -k & 0 \\ -k & k & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad (3)$$

Now, if the node 3 is fixed, than the Eq. (3) can be written as

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \bar{m} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (4)$$

Hence Eq.4 is the equations of motions of a spring and dashpot connected in series mass is in between the two. Hence the gyromass of the core system of the type two models can be converted into the simple mass by fixing a node of the core system. This system can be shown in diagram as shown in Fig. B.2.

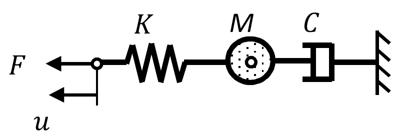

Fig. B.2. Transformed Core system of the Type II model.

Appendix-c

Output data analysis

The results obtained by running the program in OpenSees are in time domain. Hence further analyses of data are required to obtain the impedance functions in frequency domain. Here is a sample calculation by which we get the impedance functions on frequency domain.

Let the force, F applied to the node 1 be

$$F = |F|e^{i\omega t} \quad (5)$$

Where,
|F| = magnitude of the force applied to the node, i = imaginary quantity, t = time, ω = angular acceleration
Now the displacement of the node is

$$u = |u|e^{(i(\omega t - \emptyset))} \quad (6)$$

Where,
|u|= displacement magnitude, ∅ = phase lag
According the definition of the impedance function, K*

$$F = K^* u \quad (7)$$

Substituting Eq. (1) and (2) into Eq. (3),

$$K^* = \frac{|F|}{|u|} e^{i\emptyset} \quad (8)$$

Expanding Eq. (4) by using Euler's formula,

$$K^* = \frac{|F|}{|u|} \cos \emptyset + i \frac{|F|}{|u|} \sin \emptyset \quad (9)$$

The impedance function contains a real part and an imaginary part as shown in eq. (5). The real part represents the stiffness and the imaginary part represents damping of the system.

References:

[1] M. Saitoh, “Simple model of frequency-dependent impedance functions in soil-structure interaction using frequency-independent elements,” *J. Eng. Mech.*, vol. 133, no. 10, pp. 1101–1114, 2007.

[2] M. Saitoh, “On the performance of lumped parameter models with gyro-mass elements for the impedance function of a pile-group supporting a single-degree-of-freedom system,” *Earthq. Eng. Struct. Dyn.*, vol. 41, no. 4, pp. 623–641, 2012.